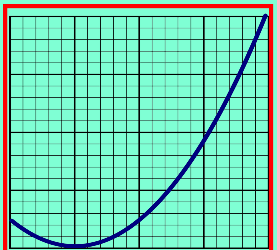
Math 125

Fall 2021

Lecture 41



$y = ax^2 + bx + c$

1) Evaluate $\begin{vmatrix} -5 & 1 \\ -2 & -7 \end{vmatrix} = -5(-7) - (-2)(1)$

$= 35 + 2 = \boxed{37}$

2) Solve By Cramer's Rule:

$$\begin{cases} 2x = 3y + 2 \\ 5x = 51 - 4y \end{cases} \Rightarrow \begin{cases} 2x - 3y = 2 \\ 5x + 4y = 51 \end{cases}$$

$D = \begin{vmatrix} 2 & -3 \\ 5 & 4 \end{vmatrix}$ $\quad D_x = \begin{vmatrix} 2 & -3 \\ 51 & 4 \end{vmatrix}$ $\quad D_y = \begin{vmatrix} 2 & 2 \\ 5 & 51 \end{vmatrix}$

$D = 2(4) - 5(-3) = 8 + 15 = \boxed{23}$

$D_x = 2(4) - (-3)(51) = 8 + 153 = \boxed{161}$

$D_y = 2(51) - 5(2) = 102 - 10 = \boxed{92}$

$x = \frac{D_x}{D} = \frac{161}{23} = \boxed{7}$ $\quad y = \frac{D_y}{D} = \frac{92}{23} = \boxed{4}$

Final Ans
(7, 4)

Evaluate $\begin{vmatrix} \boxed{2 \quad -4 \quad 2} \\ -1 \quad\; 0 \quad\; 5 \\ 3 \quad\; 0 \quad\; 4 \end{vmatrix}$

Expand by first row.

$$= 2\begin{vmatrix} 0 & 5 \\ 0 & 4 \end{vmatrix} - (-4)\begin{vmatrix} -1 & 5 \\ 3 & 4 \end{vmatrix} + 2\begin{vmatrix} -1 & 0 \\ 3 & 0 \end{vmatrix}$$

$$= 2(0) + 4(-4-15) + 2(0)$$

$$= 4(-19) = \boxed{-76}$$

Solve for y only using Cramer's rule:

$$\begin{cases} x + y + z = 4 \\ x - 2y + z = 7 \\ x + 3y + 2z = 4 \end{cases}$$

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 3 & 2 \end{vmatrix}$$

$$= 1\begin{vmatrix} -2 & 1 \\ 3 & 2 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} + 1\begin{vmatrix} 1 & -2 \\ 1 & 3 \end{vmatrix}$$

$$= 1(-4-3) - 1(2-1) + 1(3+2)$$

$$= -7 - 1 + 5 = \boxed{-3}$$

$$D_y = \begin{vmatrix} 1 & 4 & 1 \\ 1 & 7 & 1 \\ 1 & 4 & 2 \end{vmatrix}$$

$$= 1\begin{vmatrix} 7 & 1 \\ 4 & 2 \end{vmatrix} - 4\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} + 1\begin{vmatrix} 1 & 7 \\ 1 & 4 \end{vmatrix}$$

$$= 10 - 4 - 3 = \boxed{3}$$

$$y = \frac{D_y}{D} = \frac{3}{-3} = \boxed{-1}$$

Solve by matrix method: Augmented Matrix

$$\begin{cases} 3x - 4y + 4z = 7 \\ x - y - 2z = 2 \\ 2x - 3y + 6z = 5 \end{cases} \qquad \left[\begin{array}{ccc|c} 3 & -4 & 4 & 7 \\ 1 & -1 & -2 & 2 \\ 2 & -3 & 6 & 5 \end{array}\right]$$

$R1 \leftrightarrow R2$

$$\left[\begin{array}{ccc|c} 1 & -1 & -2 & 2 \\ 3 & -4 & 4 & 7 \\ 2 & -3 & 6 & 5 \end{array}\right]$$

$(-3)R1 + R2 \to R2$ $(-2)R1 + R3 \to R3$

$$\left[\begin{array}{ccc|c} 1 & -1 & -2 & 2 \\ 0 & -1 & 10 & 1 \\ 0 & -1 & 10 & 1 \end{array}\right]$$

$(-1)R2 + R3 \to R3$

$$\left[\begin{array}{ccc|c} 1 & -1 & -2 & 2 \\ 0 & -1 & 10 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

infinite # of solutions

Solve & check:

$$x = \sqrt{3x+7} - 3$$

$$x + 3 = \sqrt{3x+7}$$

$$(x+3)^2 = \left(\sqrt{3x+7}\right)^2$$

$$(x+3)(x+3) = 3x+7$$

$$x^2 + 3x + 3x + 9 - 3x - 7 = 0$$

$$x^2 + 3x + 2 = 0$$

$$(x+2)(x+1) = 0$$

$\boxed{x = -2}$ ✓ Checked $\qquad \boxed{x = -1}$ ✓ Checked

$$\{-2, -1\}$$

Solve and check

$\sqrt{x-4} + \sqrt{x+4} = 4$

$\sqrt{x-4} = 4 - \sqrt{x+4}$

$(\sqrt{x-4})^2 = (4-\sqrt{x+4})^2$

$x-4 = (4-\sqrt{x+4})(4-\sqrt{x+4})$

$x-4 = 16 - 4\sqrt{x+4} - 4\sqrt{x+4} + (\sqrt{x+4})^2$

$\cancel{x} - 4 = 16 - 8\sqrt{x+4} + \cancel{x} + 4$

$-4 - 20 = -8\sqrt{x+4}$

$-24 = -8\sqrt{x+4}$

Divide by -8

$3 = \sqrt{x+4}$

$(3)^2 = (\sqrt{x+4})^2$

$9 = x+4$

$\boxed{x=5}$ ✓ $\{5\}$

$\sqrt{5-4} + \sqrt{5+4} = 4$

$1 + 3 = 4$

$4 = 4$ ✓

Divide and Simplify in $a+bi$ form:

$$\frac{7+4i}{2-5i} \cdot \frac{(2+5i)}{(2+5i)}$$

$$= \frac{14+35i+8i+20i^2}{4+\cancel{10i}-\cancel{10i}-25i^2} = \frac{14+43i-20}{4+25}$$

$$= \frac{-6+43i}{29} = \boxed{\frac{-6}{29} + \frac{43}{29}i}$$

Simplify

$(1+2i)(1-2i)(3-4i)(3+4i)$

$= (1 \cancel{-2i} \cancel{+2i} - 4i^2)(9 \cancel{+12i} \cancel{-12i} - 16i^2)$

$= (1+4)(9+16) = 5 \cdot 25 = \boxed{125}$

Simplify

$(2+3i)(1-i)(4+3i)$

$= (2 - 2i + 3i - 3i^2)(4+3i)$

$= (2 + i + 3)(4+3i)$

$= (5+i)(4+3i) = 20 + 15i + 4i + 3i^2$

$= 20 + 19i - 3$

$= \boxed{17 + 19i}$

Given: $(x+3)^2 + (y-4)^2 = 9$

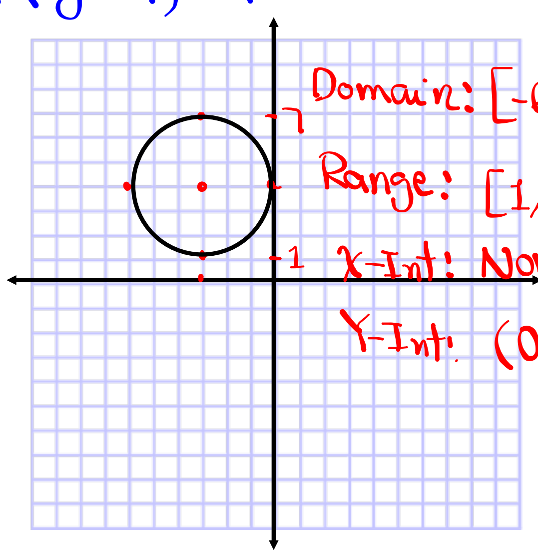
1) Center $(-3,4)$

2) Radius $r=3$

3) Draw

4) Domain & Range

5) All intercepts



Domain: $[-6,0]$

Range: $[1,7]$

X-Int: None

Y-Int: $(0,4)$

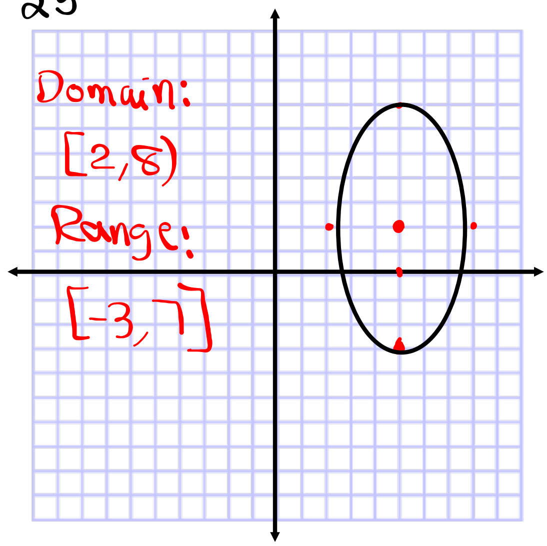
Given $\dfrac{(x-5)^2}{9} + \dfrac{(y-2)^2}{25} = 1$

1) Center $(5,2)$

2) $a^2 = 9$ $a = 3$

3) $b^2 = 25$ $b = 5$

4) Draw



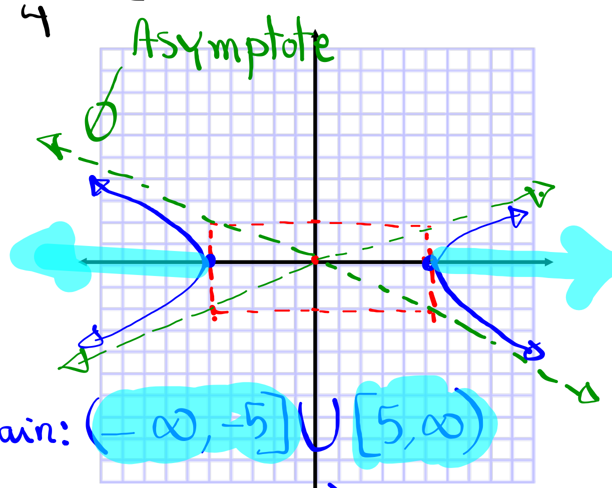
5) Domain & Range

Given $\dfrac{x^2}{25} - \dfrac{y^2}{4} = 1$

1) Center $(0,0)$

2) $a^2 = 25$ $a = 5$

3) $b^2 = 4$ $b = 2$



4) Draw

Domain: $(-\infty, -5] \cup [5, \infty)$

Range: $(-\infty, \infty)$

5) Domain & Range

x-Int: $(-5,0)$, $(5,0)$

Y-Int: None

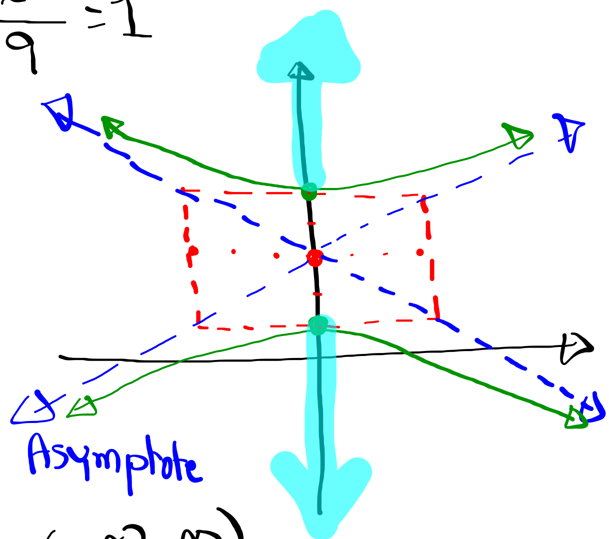
Given $\dfrac{(y-3)^2}{4} - \dfrac{x^2}{9} = 1$

1) Center $(0,3)$

2) $a^2=9 \quad a=3$

3) $b^2=4 \quad b=2$

4) Draw



Domain: $(-\infty, \infty)$

Range: $(-\infty, 1] \cup [5, \infty)$

Graph of $y = ax^2 + bx + c$ contains $(1,-5)$, $(-1,-11)$, and $(2,0)$. Find a, b, and c.

$(1,-5) \Rightarrow x=1,\ y=-5$

$-5 = a(1)^2 + b(1) + c \Rightarrow \boxed{a+b+c=-5}$

$(-1,-11) \Rightarrow x=-1,\ y=-11$

$-11 = a(-1)^2 + b(-1) + c \Rightarrow \boxed{a-b+c=-11}$

$(2,0) \Rightarrow x=2,\ y=0$

$0 = a(2)^2 + b(2) + c \Rightarrow \boxed{4a+2b+c=0}$

$$\begin{cases} a+b+c=-5 \\ a-b+c=-11 \\ 4a+2b+c=0 \end{cases}$$

$$\begin{aligned} &\begin{cases} a+b+c=-5 \\ -1\,\}\ a-b+c=-11 \end{cases} \\ &\overline{\quad 2b = 6 \quad} \quad \boxed{b=3} \end{aligned}$$

$a+3+c=-5$

$4a+2(3)+c=0$

$\Rightarrow -1\begin{cases} a+c=-8 \\ 4a+c=-6 \end{cases} \Rightarrow \begin{cases} -a-c=8 \\ 4a+c=-6 \end{cases}$

$3a = 2$

$\boxed{a=2/3}$

$\frac{2}{3}+c=-8 \qquad c=-8-\frac{2}{3}=\boxed{\frac{-26}{3}}$

I have 6 Coins
Nickel, Dimes, Quarters

Quarters = Sum of # Nickels and # Dimes

Total value $1.

How many of each?

$Q + D + N = 6$

$Q = D + N$

$\underbrace{25Q + 10D + 5N = 100}_{\text{Divide by } 5}$

$$\begin{cases} Q + D + N = 6 \\ Q - D - N = 0 \\ 5Q + 2D + N = 20 \end{cases}$$

$2Q = 6$

$\boxed{Q = 3}$

$$+\begin{cases} D + N = 3 \\ 2D + N = 5 \end{cases} \Rightarrow \boxed{D = 2} \quad \boxed{N = 1}$$

3 Quarters
2 Dimes
1 Nickel